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Non-supersymmetric black rings as thermally excited supertubes

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Abstract: We construct a seven-parameter family of supergravity solutions that describe non-supersymmetric black rings and black tubes with three charges, three dipoles and two angular momenta. The black rings have regular horizons and non-zero temperature. They are naturally interpreted as the supergravity descriptions of thermally excited configurations of supertubes, specifically of supertubes with two charges and one dipole, and of supertubes with three charges and two dipoles. In order to fully describe thermal excitations near supersymmetry of the black supertubes with three charges and three dipoles a more general family of black ring solutions is required.

Keywords: D-branes, Black Holes in String Theory, Black Holes.

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Contents

1. Introduction

Black rings are a fascinating outcome of recent studies of higher-dimensional gravity. They show that several classic results of black hole theory cannot be generalized to five dimensions: black rings have non-spherical horizon topology $S^1 \times S^2$, and their mass and spin are insufficient to fully distinguish between them and between other black holes of spherical topology [1, 2].

The remarkable progress in the string theory description of black holes had not hinted at the existence of black rings. So, initially, black rings appeared to be uncalled-for objects, and their role in string theory was unclear. A step to improve the understanding of black rings in string theory was taken in ref. [3] (following [4]), where a connection was found between black rings with two charges and another class of objects of recent interest in string theory, the so-called supertubes [5, 6]. More recently this connection has been significantly strengthened and extended with the discovery in [7] of a supersymmetric black ring of five-dimensional supergravity, with a regular horizon of finite area. This has prompted the further study of supersymmetric rings, including the generalization to three-charge solutions [8, 9, 10] and other extensions and applications [11]–[15]. The authors of [15] have actually succeeded in providing a statistical counting of their Bekenstein-Hawking entropy. It is naturally interesting to try to extend these results to include near-supersymmetric black rings.

In this paper, we present a seven-parameter family of non-supersymmetric black ring solutions which generalize the ones studied in ref. [3]. The new solutions describe black rings with three conserved charges, three dipole charges, two unequal angular momenta, and finite energy above the BPS bound. We are motivated by the wish to understand the microscopic nature of the thermal excitations of two- and three-charge supertubes. We argue that the near-supersymmetric limits of the black rings in this paper can be interpreted as thermally excited supertubes with two charges and one dipole, or thermally excited supertubes with three charges and two dipoles. As a further motivation, note that — contrary to spherical black holes — the black rings carry non-conserved charges (the dipole charges). As such the non-supersymmetric black ring solutions provide an exciting laboratory for examining new features of black holes, for instance the appearance of the non-conserved charges in the first law of black hole thermodynamics [2, 16].

We find the non-supersymmetric black rings by solution-generating techniques (boosts and U-dualities). This was also the approach in [3, 4], where the neutral five-dimensional black ring was first uplifted to six dimensions, to become a black tube.[1] Then a sequence of solution-generating transformations yielded new two-charge black tube solutions with the same charges as a supertube [5, 6]. In the supersymmetric limit the area of these black tubes vanishes and one recovers the supergravity description of a two-charge supertube [6]. Thus these charged black rings can be regarded as the result of thermally exciting a supertube. A limitation of the charged black rings built in [3] is that their supersymmetric limit could only yield supertubes with half the maximum value of the angular momentum, instead

[1] Throughout this paper we refer to the same object as a ring (in the five-dimensional description) or as a tube, when lifted to six or more dimensions.

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of the whole range of angular momenta that supertubes can have. This shortcoming is automatically resolved in this paper. The additional parameters in our new solutions allow us to construct thermal deformations for supertubes with angular momenta covering precisely the entire physically permitted range.

The extra parameters in our solutions come from choosing a more general seed solution to which the generating transformations are applied. While in [3] the seed solution was the neutral black tube, we here use the dipole black tubes of [2] as seed solutions.

The additional degrees of freedom in the dipole solutions in fact allow us to construct non-supersymmetric black rings with three charges and three dipoles. Spherical five-dimensional black holes with three charges are the most thoroughly studied black holes in string theory [17], so having three-charge black rings should be instrumental to develop the proposal in ref. [3] for a microscopic understanding of non-uniqueness and non-spherical topologies. A different motivation to further study D1-D5-P configurations is Mathur's programme to identify string microstates as non-singular horizonless solutions [18]. In fact this led the authors of [19, 20] to independently conjecture the existence of supersymmetric black rings.

The supersymmetric limit of our solutions can only reproduce a supersymmetric ring with three charges and at most two dipoles. The complete three-charge/three-dipole black rings presented in [8, 9, 10], and the minimal supersymmetric ring of [7], are not limits of the solutions in this paper. Indeed, this becomes obvious by simply counting parameters. The supersymmetric rings of [8, 9, 10] have seven independent parameters. This is the same number as in the solutions in this paper, but in the latter, one of the parameters measures the deviation away from supersymmetry. It appears that in order to find the appropriately larger family of non-supersymmetric black ring solutions one should start with a more general seed, presumably a dipole black ring with two independent angular momenta. Nevertheless, the solutions we present here seem to be adequate to describe thermal excitations near supersymmetry of supertubes with two charges and one dipole, and of some supertubes with three charges and two dipoles.

The rest of the paper is organized as follows: in the next section we discuss the general problem of how to construct the solutions, and then present them and compute their main physical properties. In section 3 we analyze the extremal and supersymmetric limits of these black rings. Section 4 analyzes the particular case of solutions to minimal five-dimensional supergravity. In section 5 we study black rings as thermally excited D1-D5-P supertubes, and consider in particular the cases of tubes with two charges and one dipole, and three charges and two dipoles. We also study the decoupling limit. We conclude in section 6 with a discussion of the consequences of our results. In appendices A and B we provide the details for the form fields in the solutions, and in appendices C and D we study the limits where the solutions reproduce spherical rotating black holes (at zero radius), and black strings (at infinite radius).

2. Non-supersymmetric black rings with three charges

In this section we first describe the sequence of boosts and dualities that we exploit to generate the charged black ring solutions. The idea is to follow the same path that yielded the

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three-charge rotating black hole [21]–[24], but this procedure becomes quite more complex and subtle when applied to black rings. We then present the solution in its most symmetric form, as an eleven dimensional supergravity supertube with three M2 charges and three M5 dipole charges, and analyze its structure and physical properties.

2.1 Generating the solution

We begin by reviewing the process followed in [3] to obtain two-charge black rings, and why a problem arises when trying to add a third charge. Starting from a five-dimensional neutral black ring that rotates along the direction ψ, add to it a flat direction z to build a six-dimensional black tube. This is then embedded into IIB supergravity by further adding four toroidal directions z_1, z_2, z_3, z_4 (which will play little more than a spectator role in the following). Now submit the solution to the following sequence of transformations: IIB S-duality; boost along z with rapidity parameter α_1; T-duality along z; boost along z with parameter α_2; T-duality along z_1; S-duality; T-duality along z_1, z_2, z_3, z_4; S-duality; T-duality along z; T-duality along z_1; S-duality. Schematically,[2]

$$\begin{aligned} S \to \mathrm{Boost}_{\alpha_1}(z) \to T(z) \to \mathrm{Boost}_{\alpha_2}(z) \to T(1) \to \\ \to S \to T(1234) \to S \to T(z) \to T(1) \to S\,. \end{aligned} \tag{2.1}$$

As a result we obtain a non-supersymmetric solution of IIB supergravity. The system has two net charges corresponding to D1 and D5-branes, and there is a dipole charge from a Kaluza-Klein monopole (kkm). The branes are arranged as

$$\begin{array}{rlcccccc} \alpha_1 & \text{D5:} & 1 & 2 & 3 & 4 & z & \_ \\ \alpha_2 & \text{D1:} & \_ & \_ & \_ & \_ & z & \_ \,. \\ (\alpha_1\alpha_2) & \text{kkm:} & 1 & 2 & 3 & 4 & (z) & \psi \end{array} \tag{2.2}$$

Following ref. [8], we use uppercase letters to denote brane components with net conserved charges (D1, D5), and lowercase for dipole brane charges (kkm). If the parameters on the left are set to zero then the corresponding brane constituent disappears, e.g.,when either of the α_i is zero the corresponding D-brane is absent, and the kkm dipole, which is fibered in the z direction, vanishes. Note that the dipole is induced as a result of charging up the tube, and would not be present if instead one charged up a spherical black hole. Ref. [3] argued that these black rings describe thermally excited supertubes: configurations where D1 and D5 branes are 'dissolved' in the worldvolume of a tubular KK monopole.

It is natural in this context to try to have a third charge on the ring, coming from momentum P propagating along the tube direction z. In order to endow the system with this charge, one might try to perform a third boost on the solution. However, as discussed in [3], the kkm fibration along the direction z is incompatible with such a boost. A naive application of a boost transformation to the solution results into a globally ill-defined geometry with Dirac-Misner string singularities (a geometric analogue of Dirac strings, to be discussed in detail later in this section).

[2] The first and last S-dualities, and the two T(1) dualities, are introduced simply to have both the initial and final solutions later in this subsection as configurations of a D1-D5 system.

To overcome this problem, in this paper we choose to start from a different seed solution which already contains three dipole charges, with parameters μ_i, but no net conserved charges [2]. Beginning now from a black tube with dipole charges d1, d5 and kkm given as

$$\begin{array}{lllllllll} \mu_1 & \text{kkm:} & 1 & 2 & 3 & 4 & (z) & \psi & \\ \mu_2 & \text{d5:} & 1 & 2 & 3 & 4 & \_ & \psi\,, & \\ \mu_3 & \text{d1:} & \_ & \_ & \_ & \_ & \_ & \psi & \end{array} \tag{2.3}$$

and acting with the sequence (2.1) followed by a boost α_3 along z, we obtain a black tube with the same dipole charges but now also D1-D5-P net charges. The branes are arranged as

$$\begin{array}{rrllllll} \alpha_1 & \text{D5:} & 1 & 2 & 3 & 4 & z & \_ \\ \alpha_2 & \text{D1:} & \_ & \_ & \_ & \_ & z & \_ \\ \alpha_3 & \text{P:} & \_ & \_ & \_ & \_ & z & \_ \\ (\alpha_2\alpha_3),\ \mu_1 & \text{d1:} & \_ & \_ & \_ & \_ & \_ & \psi\,. \\ (\alpha_1\alpha_3),\ \mu_2 & \text{d5:} & 1 & 2 & 3 & 4 & \_ & \psi \\ (\alpha_1\alpha_2),\ \mu_3 & \text{kkm:} & 1 & 2 & 3 & 4 & (z) & \psi \end{array} \tag{2.4}$$

We will show that by appropriately choosing the parameters of the solution we can manage to eliminate the global pathologies produced by having boosted the Kaluza-Klein monopoles along their fiber directions. Roughly speaking, the pathology from boosting the kkm dipole induced by charging up the solution is cancelled against the pathologies from boosting the dipoles of the initial configuration (2.3). To this effect, a single dipole in the seed solution would be sufficient, but using the complete solution (2.3) we will obtain a larger family of charged black rings.

There is a more symmetrical M-theory version of the solutions, obtained by performing $\to T(34) \to T(z)$ and then uplifting to eleven dimensions. This configuration is

$$\begin{array}{rrlllllll} \alpha_1 & \text{M2:} & 1 & 2 & \_ & \_ & \_ & \_ & \_ \\ \alpha_2 & \text{M2:} & \_ & \_ & 3 & 4 & \_ & \_ & \_ \\ \alpha_3 & \text{M2:} & \_ & \_ & \_ & \_ & 5 & 6 & \_ \\ (\alpha_2\alpha_3),\ \mu_1 & \text{m5:} & \_ & \_ & 3 & 4 & 5 & 6 & \psi\,, \\ (\alpha_1\alpha_3),\ \mu_2 & \text{m5:} & 1 & 2 & \_ & \_ & 5 & 6 & \psi \\ (\alpha_1\alpha_2),\ \mu_3 & \text{m5:} & 1 & 2 & 3 & 4 & \_ & \_ & \psi \end{array} \tag{2.5}$$

where we have set $z_5 \equiv z$, and the eleventh dimensional direction is z_6. It is this form of the solution that we present next.

2.2 Solution

The metric for the eleven-dimensional solution is

$$\begin{aligned} ds^2_{11\mathrm{D}} = ds^2_{5\mathrm{D}} &+ \left[\frac{1}{h_1}\frac{H_1(y)}{H_1(x)}\right]^{2/3}\left[h_2\,h_3\,\frac{H_2(x)H_3(x)}{H_2(y)H_3(y)}\right]^{1/3}(dz_1^2+dz_2^2)+ \\ &+\left[\frac{1}{h_2}\frac{H_2(y)}{H_2(x)}\right]^{2/3}\left[h_1\,h_3\,\frac{H_1(x)H_3(x)}{H_1(y)H_3(y)}\right]^{1/3}(dz_3^2+dz_4^2)+ \\ &+\left[\frac{1}{h_3}\frac{H_3(y)}{H_3(x)}\right]^{2/3}\left[h_1\,h_2\,\frac{H_1(x)H_2(x)}{H_1(y)H_2(y)}\right]^{1/3}(dz_5^2+dz_6^2)\,, \end{aligned} \tag{2.6}$$

where

$$
\begin{aligned}
ds^2_{5\mathrm{D}} \;=\; & -\frac{1}{(h_1h_2h_3)^{2/3}}\frac{H(x)}{H(y)}\frac{F(y)}{F(x)}\Big(dt+\omega_\psi(y)d\psi+\omega_\phi(x)d\phi\Big)^2+ \\
& +(h_1h_2h_3)^{1/3}F(x)H(x)H(y)^2\times \\
& \times\frac{R^2}{(x-y)^2}\left[-\frac{G(y)}{F(y)H(y)^3}d\psi^2-\frac{dy^2}{G(y)}+\frac{dx^2}{G(x)}+\frac{G(x)}{F(x)H(x)^3}d\phi^2\right], \qquad (2.7)
\end{aligned}
$$

and the three-form potential is

$$
\mathcal{A}=A^1\wedge dz_1\wedge dz_2+A^2\wedge dz_3\wedge dz_4+A^3\wedge dz_5\wedge dz_6\,. \qquad (2.8)
$$

The explicit expressions for the components of the one-forms A^i $(i=1,2,3)$ are given in appendix A.

We have defined the following functions

$$
\begin{aligned}
F(\xi) &= 1+\lambda\xi\,, & G(\xi) &= (1-\xi^2)(1+\nu\xi)\,,\\
H_i(\xi) &= 1-\mu_i\xi\,, & H(\xi) &\equiv [H_1(\xi)H_2(\xi)H_3(\xi)]^{1/3}\,,
\end{aligned} \qquad (2.9)
$$

and

$$
h_i=c_i{}^2-U_is_i{}^2\,, \qquad (2.10)
$$

where the functions U_i are defined in (A.10) and, in order to reduce notational clutter, we have introduced

$$
c_i\equiv\cosh\alpha_i\,,\qquad s_i\equiv\sinh\alpha_i\,. \qquad (2.11)
$$

It is useful to give explicit expressions for the h_i:

$$
\begin{aligned}
h_1 \;=\; 1+\frac{H_1(y)s_1^2}{H_1(x)F(x)H(y)^3}\Big[&(\lambda-\mu_1+\mu_2+\mu_3)(x-y)-(\mu_2\mu_3+\lambda\mu_1)(x^2-y^2)+\\
&+(\mu_1\mu_2\mu_3+\lambda\mu_1\mu_2+\lambda\mu_1\mu_3-\lambda\mu_2\mu_3)\,xy(x-y)\Big], \qquad (2.12)
\end{aligned}
$$

and h_2, h_3 obtained by exchanging $1\leftrightarrow 2$ and $1\leftrightarrow 3$, respectively.

The components of the one-form $\omega=\omega_\psi\,d\psi+\omega_\phi\,d\phi$ are

$$
\begin{aligned}
\omega_\psi(y) &= R(1+y)\left[\frac{C_\lambda}{F(y)}c_1c_2c_3-\frac{C_1}{H_1(y)}c_1s_2s_3-\frac{C_2}{H_2(y)}s_1c_2s_3-\frac{C_3}{H_3(y)}s_1s_2c_3\right], \qquad (2.13)\\
\omega_\phi(x) &= -R(1+x)\left[\frac{C_\lambda}{F(x)}s_1s_2s_3-\frac{C_1}{H_1(x)}s_1c_2c_3-\frac{C_2}{H_2(x)}c_1s_2c_3-\frac{C_3}{H_3(x)}c_1c_2s_3\right], \qquad (2.14)
\end{aligned}
$$

where

$$
C_\lambda=\epsilon_\lambda\sqrt{\lambda(\lambda-\nu)\frac{1+\lambda}{1-\lambda}}\,,\qquad C_i=\epsilon_i\sqrt{\mu_i(\mu_i+\nu)\frac{1-\mu_i}{1+\mu_i}} \qquad (2.15)
$$

for $i=1,2,3$. A choice of sign $\epsilon_i,\epsilon_\lambda=\pm1$ has been included explicitly.

We assume that the coordinates z_i, $i=1,\dots,6$ are periodically identified. The coordinates x and y take values in the ranges

$$
-1\le x\le 1\,,\qquad -\infty<y\le -1\,,\qquad \frac{1}{\min\mu_i}<y<\infty\,. \qquad (2.16)
$$

The solution has three Killing vectors, ∂_t, ∂_ψ, and ∂_ϕ, and is characterized by eight dimensionless parameters $\lambda, \nu, \mu_i, \alpha_i$, plus the scale parameter R, which has dimension of length.

Without loss of generality we can take $R > 0$. The parameters λ, ν, μ_i are restricted as

$$0 < \nu \leq \lambda < 1 \,, \qquad 0 \leq \mu_i < 1 \,, \tag{2.17}$$

while the α_i can initially take any real value. These ranges of values are typically sufficient to avoid the appearance of naked curvature singularities. Below we will discuss how the elimination of other pathologies will reduce the total number of free parameters from nine to seven.

Each α_i is associated with an M2-brane charge; taking $\alpha_i = 0$ sets the corresponding M2-brane charge to zero. In particular, taking all $\alpha_i = 0$, we recover the dipole black rings of [2]. The solutions contain contributions to the M5-brane dipole charges that originate both from the parameters μ_i as well as from the boosts, see (2.5). The precise relation between the parameters and the charges will be given below.

Asymptotic infinity is at $x, y \to -1$. Since $x = -1$ and $y = -1$ are fixed point sets of respectively ∂_ϕ and ∂_ψ, the periodicites of ψ and ϕ must be chosen so as to avoid conical defects that would extend to infinity. The required periodicities are

$$\Delta\psi = \Delta\phi = 2\pi \frac{\sqrt{1-\lambda}}{1-\nu} \prod_{i=1}^{3} \sqrt{1+\mu_i} \,. \tag{2.18}$$

Defining canonical angular variables

$$\tilde{\psi} = \frac{2\pi}{\Delta\psi}\, \psi \,, \qquad \tilde{\phi} = \frac{2\pi}{\Delta\phi}\, \phi \,, \tag{2.19}$$

and performing the coordinate transformation

$$\zeta_1 = \tilde{R}\frac{\sqrt{-1-y}}{x-y} \,, \qquad \zeta_2 = \tilde{R}\frac{\sqrt{1+x}}{x-y} \,, \qquad \tilde{R}^2 = 2R^2\, \frac{1-\lambda}{1-\nu} \prod_{i=1}^{3} (1+\mu_i) \,, \tag{2.20}$$

the five-dimensional asymptotic metric takes the manifestly flat form

$$ds^2_{5\mathrm{D}} = -dt^2 + d\zeta_1^2 + \zeta_1^2\, d\tilde{\psi}^2 + d\zeta_2^2 + \zeta_2^2\, d\tilde{\phi}^2 \,. \tag{2.21}$$

Thus the metric is asymptotically five-dimensional Minkowski space times a six-torus.

2.2.1 Removing Dirac-Misner strings

We are interested in ring-like solutions with horizon topology $S^1 \times S^2$. In order that (x, ϕ) parameterize a two-sphere, ∂_ϕ must have fixed-points at $x = \pm 1$, corresponding to the poles of the S^2. However, note that when the three α_i are non-zero, the orbit of ∂_ϕ does not close off at $x = 1$, since $\omega_\phi(x = 1) \neq 0$. This can be interpreted as the presence of Dirac-Misner strings — a geometric analogue of Dirac strings discussed by Misner in the Taub-NUT solution [25]. Their appearance in this solution can be traced to the fact that,

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to obtain it, we have boosted along the fiber of a Kaluza-Klein monopole [3]. By analogy with the Dirac monopoles, one might try to eliminate the strings by covering the geometry with two patches, each one regular at each pole. However, the coordinate transformation in the region where the two patches overlap would require t to be periodically identified with period $\Delta t = \omega_\phi(x = 1)\Delta\phi$ (or an integer fraction of this). Closed timelike curves would then be present everywhere outside the horizon.

To remove the pathology we must therefore require that the form ω be globally well-defined, i.e.,that $\omega_\phi(x = \pm 1) = 0$. This places a constraint on the parameters of the solution of the form

$$\frac{C_\lambda}{1+\lambda} s_1 s_2 s_3 = \frac{C_1}{1-\mu_1} s_1 c_2 c_3 + \frac{C_2}{1-\mu_2} c_1 s_2 c_3 + \frac{C_3}{1-\mu_3} c_1 c_2 s_3 \,. \tag{2.22}$$

Imposing this condition, ω_ϕ can be written as

$$\omega_\phi(x) = -\frac{R(1-x^2)}{F(x)} \left[\frac{\lambda+\mu_1}{1-\mu_1} \frac{C_1}{H_1(x)} s_1 c_2 c_3 + \frac{\lambda+\mu_2}{1-\mu_2} \frac{C_2}{H_2(x)} c_1 s_2 c_3 + \frac{\lambda+\mu_3}{1-\mu_3} \frac{C_3}{H_3(x)} c_1 c_2 s_3 \right], \tag{2.23}$$

which is manifestly regular.

2.2.2 Balancing the ring

The choice (2.18) for the period of ϕ makes the orbits of ∂_ϕ close off smoothly at $x = -1$. We have also required that the orbits of ∂_ϕ close off at the other pole, $x = +1$, by imposing the condition (2.22), but there still remains the possibility that conical defects are present at this pole. Smoothness at $x = 1$ requires another, specific value for $\Delta\phi$, and it is easy to see that this is compatible with (2.18) only if the parameters satisfy the equation

$$\left(\frac{1-\nu}{1+\nu}\right)^2 = \frac{1-\lambda}{1+\lambda} \prod_{i=1}^{3} \frac{1+\mu_i}{1-\mu_i} \,. \tag{2.24}$$

Violating this condition results in a disk-like conical singularity inside the ring at $x = 1$. Depending on whether there is an excess or deficit angle, the disk provides a push or pull to keep the ring in equilibrium. Thus (2.24) is a balancing condition. We assume the ring is balanced, i.e.,that (2.24) holds. This condition is independent of α_i and hence the same condition was found for the dipole black rings in [2].

With the balancing condition (2.24) and the Dirac-Misner condition (2.22), the solution contains seven independent parameters: the scale R, plus six dimensionless parameters. These may be taken to be μ_i and α_i, if λ and ν are eliminated through (2.22) and (2.24).

2.3 Properties

We give here expressions for the conserved charges (mass, angular momentum and net charge) as well as for the dipole charges. We then analyze the horizon geometry and compute the horizon area, temperature and angular velocity of the black rings.

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2.3.1 Asymptotic charges

If we assume that the z_i directions are all compact with period $2\pi\ell$ then the five-dimensional Newton's constant is related to the 11D coupling constant κ through $\kappa^2 = 8\pi G_5 (2\pi\ell)^6$. Also, note that the six-torus parametrized by the z_i has constant volume. This constraint implies that the five-dimensional metric $ds^2_{5\text{D}}$ is the same as the Einstein-frame metric arising from the reduction of the eleven-dimensional metric (2.6) on the T^6. The mass and angular momenta in five dimensions can then be obtained from the asymptotic form of the metric.

The mass is most simply expressed as

$$M = \frac{\pi}{4G_5}\Big(Q_1 \coth 2\alpha_1 + Q_2 \coth 2\alpha_2 + Q_3 \coth 2\alpha_3\Big)\,, \tag{2.25}$$

in terms of the M2-brane charges carried by the solution,

$$\begin{aligned}
Q_1 &= \frac{R^2 \sinh 2\alpha_1}{1-\nu}\Big[\lambda - \mu_1 + \mu_2 + \mu_3 + 2(\mu_2\mu_3 + \lambda\mu_1) + \\
&\qquad\qquad + \lambda(\mu_1\mu_2 + \mu_1\mu_3 - \mu_2\mu_3) + \mu_1\mu_2\mu_3\Big]\,, \\
Q_2 &= \frac{R^2 \sinh 2\alpha_2}{1-\nu}\Big[\lambda + \mu_1 - \mu_2 + \mu_3 + 2(\mu_1\mu_3 + \lambda\mu_2) + \\
&\qquad\qquad + \lambda(\mu_1\mu_2 - \mu_1\mu_3 + \mu_2\mu_3) + \mu_1\mu_2\mu_3\Big]\,, \\
Q_3 &= \frac{R^2 \sinh 2\alpha_3}{1-\nu}\Big[\lambda + \mu_1 + \mu_2 - \mu_3 + 2(\mu_1\mu_2 + \lambda\mu_3) + \\
&\qquad\qquad + \lambda(-\mu_1\mu_2 + \mu_1\mu_3 + \mu_2\mu_3) + \mu_1\mu_2\mu_3\Big]\,,
\end{aligned} \tag{2.26}$$

and satisfies the BPS bound

$$M \geq \frac{\pi}{4G_5}\Big(\,|Q_1| + |Q_2| + |Q_3|\Big)\,. \tag{2.27}$$

The two angular momenta are

$$\begin{aligned}
J_\psi &= \frac{\pi R^3}{2G_5}\frac{(1-\lambda)^{3/2}}{(1-\nu)^2}\left[\prod_{i=1}^{3}(1+\mu_i)^{3/2}\right] \times \\
&\quad \times \left[\frac{C_\lambda}{1-\lambda}c_1c_2c_3 - \frac{C_1}{1+\mu_1}c_1s_2s_3 - \frac{C_2}{1+\mu_2}s_1c_2s_3 - \frac{C_3}{1+\mu_3}s_1s_2c_3\right],
\end{aligned} \tag{2.28}$$

$$\begin{aligned}
J_\phi &= -\frac{\pi R^3}{2G_5}\frac{(1-\lambda)^{3/2}}{(1-\nu)^2}\left[\prod_{i=1}^{3}(1+\mu_i)^{3/2}\right] \times \\
&\quad \times \left[\frac{C_\lambda}{1-\lambda}s_1s_2s_3 - \frac{C_1}{1+\mu_1}s_1c_2c_3 - \frac{C_2}{1+\mu_2}c_1s_2c_3 - \frac{C_3}{1+\mu_3}c_1c_2s_3\right].
\end{aligned} \tag{2.29}$$

Eq. (2.22) can be used to write the latter as

$$\begin{aligned}
J_\phi = -\frac{\pi R^3}{G_5}\frac{\sqrt{1-\lambda}}{(1-\nu)^2}\left[\prod_{i=1}^{3}(1+\mu_i)^{3/2}\right] \times \\
\times \left[\frac{\lambda+\mu_1}{1-\mu_1^2}\,C_1s_1c_2c_3 + \frac{\lambda+\mu_2}{1-\mu_2^2}\,C_2c_1s_2c_3 + \frac{\lambda+\mu_3}{1-\mu_3^2}\,C_3c_1c_2s_3\right],
\end{aligned} \tag{2.30}$$

although it does not lead to any simpler expressions for J_ψ or Q_i.

2.3.2 Dipole charges

The dipole charges are given by

$$q_i = \frac{1}{2\pi(2\pi\ell)^2}\int_{S^2\times T^2} d\mathcal{A} = \frac{1}{2\pi}\int_{S^2} dA^i = \frac{\Delta\phi}{2\pi}\left[A^i_\phi(x=1) - A^i_\phi(x=-1)\right], \tag{2.31}$$

where the two-sphere parameterized by (x,ϕ) surrounds a constant-ψ slice of the black ring and for $i=1,2,3$ the two-torus is parameterized by z_1-z_2, z_3-z_4 or z_5-z_6, respectively. For generic values of the parameters the dipole charges are not well-defined since the expressions (2.31) are y-dependent. The condition for the corresponding gauge fields to be well-defined is the same as imposing the absence of Dirac-Misner strings (2.22). With this, the y-dependence drops out and we find

$$q_1 = -\frac{2R}{s_1}\frac{\sqrt{1-\lambda}}{1-\nu}\left[\prod_{i=1}^{3}\sqrt{1+\mu_i}\right]\left[\frac{C_2}{1-\mu_2}s_2c_3 + \frac{C_3}{1-\mu_3}c_2s_3\right], \tag{2.32}$$

$$q_2 = -\frac{2R}{s_2}\frac{\sqrt{1-\lambda}}{1-\nu}\left[\prod_{i=1}^{3}\sqrt{1+\mu_i}\right]\left[\frac{C_1}{1-\mu_1}s_1c_3 + \frac{C_3}{1-\mu_3}c_1s_3\right], \tag{2.33}$$

$$q_3 = -\frac{2R}{s_3}\frac{\sqrt{1-\lambda}}{1-\nu}\left[\prod_{i=1}^{3}\sqrt{1+\mu_i}\right]\left[\frac{C_2}{1-\mu_2}s_2c_1 + \frac{C_1}{1-\mu_1}c_2s_1\right]. \tag{2.34}$$

One can easily verify that

$$J_\phi = \frac{\pi}{8G_5}\left(q_1Q_1 + q_2Q_2 + q_3Q_3\right). \tag{2.35}$$

This identity reflects the fact that the second angular momentum J_ϕ appears as a result of charging up the dipole rings.

2.3.3 Non-uniqueness

There are seven parameters in the solution, but only six conserved charges at infinity, $(M,\ J_\psi,\ J_\phi,\ Q_{1,2,3})$. So fixing these parameters we can expect to find a one-parameter continuous non-uniqueness.

2.3.4 Horizon

As for the dipole black rings of [2], we expect the event horizon to be located at $y = y_h \equiv -1/\nu$. At $y = y_h$, g_{yy} blows up, but this is just a coordinate singularity which can be removed by the coordinate transformation $(t,\psi)\to(v,\psi')$ given as

$$dt = dv + \omega_\psi(y)\,\frac{\sqrt{-F(y)H(y)^3}}{G(y)}dy\,, \qquad d\psi = d\psi' - \frac{\sqrt{-F(y)H(y)^3}}{G(y)}dy\,. \tag{2.36}$$

Then the five-dimensional part of the metric is

$$\begin{aligned} ds^2_{5\mathrm{D}} =& -\frac{1}{(h_1h_2h_3)^{2/3}}\frac{H(x)}{H(y)}\frac{F(y)}{F(x)}(dv + \omega_\psi(y)d\psi' + \omega_\phi(x)d\phi)^2 + \\ &+(h_1h_2h_3)^{1/3}H(x)H(y)^2F(x)\times \\ &\times\frac{R^2}{(x-y)^2}\left[-\frac{G(y)}{F(y)H(y)^3}\,d\psi'^2 - \frac{2\,d\psi' dy}{\sqrt{-F(y)H(y)^3}} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)H(x)^3}\,d\phi^2\right], \end{aligned} \tag{2.37}$$

and thus the full metric is manifestly regular at $y = y_h$.

The metric on (a spatial section of) the horizon is

$$\begin{aligned} ds_H^2 &= \frac{1}{(h_1h_2h_3)^{2/3}}\frac{H(x)}{F(x)}\frac{|F(y_h)|}{H(y_h)}\Big(\omega_\psi(y_h)d\psi' + \omega_\phi(x)d\phi\Big)^2 + \\ &\quad + (h_1h_2h_3)^{1/3}H(x)H(y_h)^2F(x)\frac{R^2}{(x-y_h)^2}\left[\frac{dx^2}{G(x)} + \frac{G(x)}{F(x)H(x)^3}\,d\phi^2\right], \end{aligned} \tag{2.38}$$

where the h_i are evaluated at y_h, but recall that they also depend on x.

In order to better understand the geometry of this horizon, let us consider first the following simpler metric,

$$\begin{aligned} ds^2 &= R_1^2\left(d\psi' + k(1-x^2)d\phi\right)^2 + R_2^2\left(\frac{dx^2}{1-x^2} + (1-x^2)d\phi^2\right) \\ &= R_1^2\left(d\psi' + k\sin^2\theta d\phi\right)^2 + R_2^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \end{aligned} \tag{2.39}$$

where the second expression is obtained by making $x = \cos\theta$, and R_1, R_2, k, are constants. This is topologically $S^1\times S^2$. Due to the cross-term $g_{\psi'\phi}$ the product is twisted,[3] but, since $g_{\psi'\phi}$ vanishes at $\theta = 0, \pi$, i.e.,at $x = \pm 1$, the fibration of the S^1 over the S^2 is topologically trivial and globally well-defined.

The horizon metric (2.38) describes a geometry topologically equivalent to (2.39) (recall that $G(x), \omega_\phi(x) \propto (1-x^2)$), but now R_1, R_2, k are functions of the polar coordinate $x\in[-1,1]$, everywhere regular and non-vanishing. So the horizon of these black rings is topologically $S^1\times S^2$, but the radii of the S^1 and S^2, and the twisting, are not constant but change with the latitude of the S^2.

The horizon area is

$$\begin{aligned} \mathcal{A}_H &= 8\pi^2R^3\,\frac{(1-\lambda)(\lambda-\nu)^{1/2}}{(1-\nu)^2(1+\nu)}\left[\prod_{i=1}^3(1+\mu_i)(\nu+\mu_i)^{1/2}\right]\times \\ &\quad\times\left|\frac{C_\lambda}{\lambda-\nu}c_1c_2c_3 + \frac{C_1}{\nu+\mu_1}c_1s_2s_3 + \frac{C_2}{\nu+\mu_2}s_1c_2s_3 + \frac{C_3}{\nu+\mu_3}s_1s_2c_3\right|. \end{aligned} \tag{2.40}$$

When all μ_i are nonzero, there is also an inner horizon at $y=-\infty$. At $y = 1/\min\mu_i$ there is a curvature singularity hidden behind the horizons. The Killing vector ∂_t becomes spacelike at $y=-1/\lambda$, so $-1/\nu < y < -1/\lambda$ is the ergoregion. The ergosurface at $y=-1/\lambda$ has topology $S^1\times S^2$.

The horizon is generated by the orbits of the Killing vector $\boldsymbol{\xi} = \partial/\partial t - \Omega_\psi\,\partial/\partial\tilde\psi$ (where $\tilde\psi$ is the angle in (2.19)), with the angular velocity of the horizon given by

$$\begin{aligned} \Omega_\psi^{-1} &= \frac{\Delta\psi}{2\pi}\,\omega_\psi(y_h) \\ &= R\sqrt{1-\lambda}\left[\prod_i\sqrt{1+\mu_i}\right]\times \\ &\quad\times\left|\frac{C_\lambda}{\lambda-\nu}c_1c_2c_3 + \frac{C_1}{\nu+\mu_1}c_1s_2s_3 + \frac{C_2}{\nu+\mu_2}s_1c_2s_3 + \frac{C_3}{\nu+\mu_3}s_1s_2c_3\right|. \end{aligned} \tag{2.41}$$

Note that the angular velocity in the ϕ direction vanishes even if $J_\phi \neq 0$, which is rather unusual for a non-supersymmetric solution.

[3]Note that at constant $\theta\neq 0,\pi$ one recognizes a twisted 2-torus.

The temperature, obtained from the surface gravity at the horizon, is

$$
\begin{aligned}
T_H^{-1} &= 4\pi R \frac{\sqrt{\lambda-\nu}\prod_i \sqrt{\mu_i+\nu}}{\nu(1+\nu)} \times \\
&\times \left| \frac{C_\lambda}{\lambda-\nu} c_1 c_2 c_3 + \frac{C_1}{\nu+\mu_1} c_1 s_2 s_3 + \frac{C_2}{\nu+\mu_2} s_1 c_2 s_3 + \frac{C_3}{\nu+\mu_3} s_1 s_2 c_3 \right| . \qquad (2.42)
\end{aligned}
$$

The entropy of the black ring is $S = \mathcal{A}_H/4$. We note that $T_H S$ is quite simple, but $\Omega_\psi J_\psi$ is not. It will be interesting to understand the thermodynamics of these non-supersymmetric black rings. The first law for black rings with dipole charges is currently being investigated [16].

3. Extremal and supersymmetric limits

In order to avoid possible confusion, it may be worth recalling that the extremal limit and the supersymmetric limit of a black hole solution, even if they often coincide, in general need not be the same. The extremal limit is defined as the limit where the inner and outer horizons of a black hole coincide. Then, if the horizon remains regular, its surface gravity, and hence its temperature, vanish. The supersymmetric limit, instead, is one where the limiting solution preserves a fraction of supersymmetry and saturates a BPS bound. If the horizon remains regular in the supersymmetric limit, it must be degenerate, hence extremal. But the converse is not true in general. The extremal limit of a black hole need not be supersymmetric. A familiar example of this is the Kerr-Newman solution, whose extremal limit at maximal rotation is not supersymmetric, and whose supersymmetric limit, with $M = |Q|$, cannot have a regular horizon at finite rotation.

Let us first discuss the extremal limit, since it is simpler. If $\nu = 0$, and all the μ_i are non-zero, the inner and outer horizon coincide and the ring has a degenerate horizon with zero temperature. So the extremal limit is $\nu \to 0$. One finds a regular horizon of *finite area* as long as all μ_i are non-vanishing. Such extremal solutions have proven useful in order to understand the microphysics of black rings [2]. However, for finite values of the parameters other than ν, these extremal solutions are not supersymmetric. This is clear from the fact that they do not saturate the BPS bound (2.27).

In order to saturate this bound, we see from (2.25) that we must take $|\alpha_i| \to \infty$. But before analyzing this limit, it is important to realize that the solutions in this paper cannot reproduce the most general supersymmetric rings with three charges and three dipoles in ref. [8]. A simple way to see this is by noting that the latter have

$$
J_\phi = \frac{\pi}{8G_5}\left(q_1 Q_1 + q_2 Q_2 + q_3 Q_3 - q_1 q_2 q_3\right) \quad \text{(BPS ring)}\,, \qquad (3.1)
$$

instead of (2.35). The parameter count mentioned in the introduction also leads to this conclusion. Furthermore, for the supersymmetric solutions with all three charges and three dipoles, each of the functions ω_ψ and ω_ϕ depend on *both* x and y, whereas here we have $\omega_\psi(y)$ and $\omega_\phi(x)$.

It follows that at most we can recover a supersymmetric ring with three charges and *two* dipoles. This ring does not have a regular horizon. We believe, however, that this limitation of the non-supersymmetric solutions is not fundamental, but instead is just a shortcoming of our construction starting from the seed in [2] (so far the most general seed solution available). We expect that a more general non-supersymmetric black ring solution which retains the three charges and the three dipoles in the supersymmetric limit exists.

In order to take the supersymmetric limit in such a way that three charges and two dipoles survive, take $\alpha_1, \alpha_2, \alpha_3 \to \infty$ and $\lambda, \nu, \mu_i \to 0$ such that $e^{2\alpha_1} \sim e^{2\alpha_2} \sim e^{\alpha_3}$ and $\lambda \sim \mu_3 \sim e^{-\alpha_3}$, while $\mu_1 \sim \mu_2 \sim \nu \sim (\lambda - \mu_3) \sim e^{-2\alpha_3}$. Note the latter implies that $\lambda = \mu_3 + O(e^{-2\alpha_3})$, which we shall use in the following.

These scalings are conveniently encoded by saying that in the limit we keep fixed the following quantities:

$$\lambda e^{2\alpha_1} = \frac{Q_1}{R^2}\,, \qquad \lambda e^{2\alpha_2} = \frac{Q_2}{R^2}\,, \qquad \frac{1}{2}(\lambda + \mu_1 + \mu_2 - \mu_3 + 2\lambda\mu_3)e^{2\alpha_3} = \frac{Q_3}{R^2}\,, \tag{3.2}$$

$$-\epsilon_3 \lambda e^{\alpha_3+\alpha_2-\alpha_1} = \frac{q_1}{R}\,, \qquad -\epsilon_3 \lambda e^{\alpha_3-\alpha_2+\alpha_1} = \frac{q_2}{R}\,, \tag{3.3}$$

$$(\mu_1 + \mu_2 + \nu)e^{2\alpha_3} = \frac{a^2}{R^2}\,, \tag{3.4}$$

$$\frac{1}{2}\left[\mu_1 + \mu_2 + \nu + \epsilon_\lambda\left(\epsilon_1\sqrt{\mu_1(\nu+\mu_1)} + \epsilon_2\sqrt{\mu_2(\nu+\mu_2)}\right)\right] e^{2\alpha_3} = \frac{b^2}{R^2}\,. \tag{3.5}$$

The Q_i and q_i are actually the limits of the charges and dipoles in (2.26) and (2.31). Note that now the limiting $q_{1,2}$ and $Q_{1,2}$ are not independent, but satisfy

$$q_1 Q_1 = q_2 Q_2\,. \tag{3.6}$$

Recall that the $\epsilon_\lambda, \epsilon_i$ are choices of signs, $\epsilon_\lambda, \epsilon_i = \pm 1$. We have arbitrarily chosen the boosts α_i to be positive. This then requires the sign choice $\epsilon_\lambda = \epsilon_3$ in order that the cancellation of Dirac-Misner strings (2.22) be possible. The parameters a^2 and b^2 are non-negative numbers, $a^2 \geq b^2 \geq 0$ (for any choice of the signs ϵ_λ, ϵ_i), and we shall presently see that after imposing the balancing condition and the Dirac-Misner condition, they drop out from the solution.

We demand that the supersymmetric limit is approached through a sequence of black rings which are regular (on and outside the horizon), and therefore require that they satisfy the balancing condition (2.24). In the limit, this becomes

$$2\nu = \lambda - \mu_1 - \mu_2 - \mu_3\,, \tag{3.7}$$

which can be written

$$Q_3 - q_1 q_2 = a^2\,. \tag{3.8}$$

The Dirac-Misner condition (2.22) gives

$$R^2 q_1 q_2 (Q_1 + Q_2) = Q_1 Q_2 (Q_3 - q_1 q_2 - b^2)\,. \tag{3.9}$$

Now turning to the solution, we find in the supersymmetric limit that

$$h_1 = 1 + \frac{Q_1}{2R^2}(x-y)\,, \qquad h_2 = 1 + \frac{Q_2}{2R^2}(x-y)\,, \qquad h_3 = 1 + \frac{Q_3 - q_1 q_2}{2R^2}(x-y) - \frac{q_1 q_2}{4R^2}(x^2 - y^2)\,. \tag{3.10}$$

Also, using the balancing condition (3.7), the Dirac-Misner condition (3.9), and (3.6) to rewrite the expressions, we get

$$\begin{aligned} \omega_\psi(y) &= \epsilon_\lambda \left[\frac{1}{2}(q_1+q_2)(1+y) - \frac{1}{8R^2}(y^2-1)(q_1Q_1+q_2Q_2)\right], \\ \omega_\phi(x) &= -\epsilon_\lambda \frac{1}{8R^2}(1-x^2)(q_1Q_1+q_2Q_2). \end{aligned}$$

The supersymmetric-limit metric is

$$\begin{aligned} ds^2_{11} = &-\frac{1}{(h_1h_2h_3)^{2/3}}[dt+\omega_\psi(y)d\psi+\omega_\phi(x)d\phi]^2 + \\ &+(h_1h_2h_3)^{1/3}\left\{\frac{R^2}{(x-y)^2}\left[(y^2-1)d\psi^2+\frac{dy^2}{y^2-1}+\frac{dx^2}{1-x^2}+(1-x^2)\,d\phi^2\right]+\right. \\ &\left.+\frac{1}{h_1}(dz_1^2+dz_2^2)+\frac{1}{h_2}(dz_3^2+dz_4^2)+\frac{1}{h_3}(dz_5^2+dz_6^2)\right\}. \end{aligned} \tag{3.11}$$

For the three-form potentials (given in appendix A), we find after imposing the balancing condition (3.7) and the Dirac-Misner condition (3.17)

$$A^i_t = h_i^{-1} - 1, \qquad i = 1,2,3, \tag{3.12}$$

while

$$A^i_\psi = \frac{1}{h_i}\omega_\psi - \frac{q_i}{2}(1+y), \qquad A^i_\phi = \frac{1}{h_i}\omega_\phi - \frac{q_i}{2}(1+x), \qquad i = 1,2, \tag{3.13}$$

and

$$A^3_\psi = \frac{1}{h_3}\omega_\psi, \qquad A^3_\phi = \frac{1}{h_3}\omega_\phi. \tag{3.14}$$

Choosing $\epsilon_\lambda = +1$, this matches exactly the full eleven-dimensional supersymmetric solution of [8] with $q_3 = 0$.

So, as advertised, a and b disappear from the limiting solution. However, a remnant of their presence survives in the form of two constraints on the values of the parameters. Note that since $a^2 \geq 0$, the balancing condition (3.8) gives rise to the bound

$$Q_3 \geq q_1q_2, \tag{3.15}$$

which was also found in [8]. Further, using that $0 \leq b^2 \leq a^2$, the Dirac-Misner condition (3.9) gives

$$R^2 \leq \frac{Q_1Q_2(Q_3-q_1q_2)}{q_1q_2(Q_1+Q_2)}, \tag{3.16}$$

which is more meaningfully rewritten, using (3.6), as a bound on a combination of the angular momenta,

$$\frac{4G_5}{\pi}(J_\psi - J_\phi) \leq \sqrt{\frac{Q_1Q_2}{q_1q_2}}(Q_3-q_1q_2). \tag{3.17}$$

The bound (3.16) and the constraint (3.6) are precisely the conditions found in [8] in order to avoid closed causal curves for the BPS solution. It is curious that the condition for eliminating Dirac-Misner strings in the non-supersymmetric geometry becomes precisely the same as the condition of avoiding causal pathologies in the supersymmetric solution.

The supersymmetric limit of a black ring with two charges Q_1, Q_2 and one dipole q_3 does not arise as a special case of the above limit. It must be taken in a different manner, which we present in section 5.2.

4. Non-supersymmetric black rings in minimal 5D supergravity

A particular case of interest of our solutions is obtained when the three charges and the three dipoles are set equal,

$$\alpha_1 = \alpha_2 = \alpha_3 \equiv \alpha\,, \qquad \mu_1 = \mu_2 = \mu_3 \equiv \mu\,. \tag{4.1}$$

Then the three gauge fields in (2.8) associated to each of the brane components are equal, $A^1 = A^2 = A^3$, and the moduli associated to the size of the dimensions z_i are constant. The solution then becomes a non-supersymmetric black ring of the minimal supergravity theory in five dimensions. The action for the bosonic sector of this theory is

$$I = -\frac{1}{16\pi G_5}\int \sqrt{-g}\left(R - \frac{1}{4}F^2 - \frac{1}{6\sqrt{3}}\,\epsilon^{\mu\alpha\beta\gamma\delta}A_\mu F_{\alpha\beta}F_{\gamma\delta}\right), \tag{4.2}$$

where $F = dA \equiv \sqrt{3}\,dA^i$. The form of the solution is obtained in a straightforward manner from the one in section 2.2, but since it becomes quite simpler it is worth giving explicit expressions. The metric is

$$\begin{aligned} ds^2_{5\mathrm{D}} = &-\frac{1}{h_\alpha^2(x,y)}\frac{H(x)}{H(y)}\frac{F(y)}{F(x)}\Big(dt + \omega_\psi(y)d\psi + \omega_\phi(x)d\phi\Big)^2 + h_\alpha(x,y)F(x)H(x)H(y)^2 \times \\ &\times \frac{R^2}{(x-y)^2}\left[-\frac{G(y)}{F(y)H(y)^3}\,d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)H(x)^3}\,d\phi^2\right], \end{aligned} \tag{4.3}$$

with $H(\xi) = 1 - \mu\xi$, and F and G as in (2.9). The functions h_i now simplify to

$$h_\alpha(x,y) = 1 + \frac{(\lambda+\mu)(x-y)}{F(x)H(y)}\sinh^2\alpha\,. \tag{4.4}$$

The one-form ω has components

$$\omega_\psi(y) = R(1+y)\cosh\alpha\left[\frac{C_\lambda}{F(y)}\cosh^2\alpha - \frac{3C_\mu}{H(y)}\sinh^2\alpha\right] \tag{4.5}$$

and

$$\omega_\phi(x) = -R\,\frac{1-x^2}{F(x)H(x)}\,\frac{\lambda+\mu}{1+\lambda}\,C_\lambda\sinh^3\alpha\,, \tag{4.6}$$

where C_λ and $C_\mu \equiv C_i$ are given in (2.15). To obtain (4.6) we have used the condition

$$\frac{C_\lambda}{1+\lambda}\sinh^2\alpha = \frac{3C_\mu}{1-\mu}\cosh^2\alpha \tag{4.7}$$

necessary to guarantee the absence of Dirac-Misner strings.

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The physical parameters of the solution are

$$
\begin{aligned}
M &= \frac{3\pi R^2}{4G_5}\frac{(\lambda+\mu)(1+\mu)^2}{1-\nu}\cosh 2\alpha\,, &(4.8)\\
J_\psi &= \frac{\pi R^3}{2G_5}\frac{(1-\lambda)^{3/2}(1+\mu)^{9/2}}{(1-\nu)^2}\cosh\alpha\left[\frac{C_\lambda}{1-\lambda}\cosh^2\alpha-\frac{3C_\mu}{1+\mu}\sinh^2\alpha\right], &(4.9)\\
J_\phi &= -\frac{3\pi R^3}{G_5}\frac{\sqrt{1-\lambda}\,(1+\mu)^{7/2}(\lambda+\mu)}{(1-\nu)^2(1-\mu)}C_\mu\cosh^2\alpha\sinh\alpha\,, &(4.10)\\
\mathcal{A}_H &= 8\pi^2R^3\,\frac{(1-\lambda)(\lambda-\nu)^{1/2}(1+\mu)^3(\nu+\mu)^{3/2}}{(1-\nu)^2(1+\nu)}\times\\
&\quad\times\left|\frac{C_\lambda}{\lambda-\nu}\cosh^2\alpha+\frac{3C_\mu}{\nu+\mu}\sinh^2\alpha\right|\cosh\alpha\,.\\
T_H^{-1} &= 4\pi R\frac{\sqrt{\lambda-\nu}(\mu+\nu)^{3/2}}{\nu(1+\nu)}\cosh\alpha\left|\frac{C_\lambda}{\lambda-\nu}\cosh^2\alpha+\frac{3C_\mu}{\nu+\mu}\sinh^2\alpha\right|. &(4.11)\\
\Omega_\psi^{-1} &= R\sqrt{1-\lambda}(1+\mu)^{3/2}\cosh\alpha\left|\frac{C_\lambda}{\lambda-\nu}\cosh^2\alpha+\frac{3C_\mu}{\nu+\mu}\sinh^2\alpha\right|. &(4.12)
\end{aligned}
$$

We have used the Dirac-Misner condition (2.22) to simplify only the expression for J_ϕ. The charge is obtained from the relation

$$Q=\frac{4G_5}{3\pi}M\tanh 2\alpha\,, \tag{4.13}$$

and the dipole charge is

$$
\begin{aligned}
q &= -4R\frac{\sqrt{1-\lambda}(1+\mu)^{3/2}}{(1-\nu)(1-\mu)}C_\mu\cosh\alpha\\
&= \frac{8G_5}{3\pi}\frac{J_\phi}{Q}\,. &(4.14)
\end{aligned}
$$

The solution contains five parameters, $(\lambda,\nu,\mu,\alpha,R)$, but the two constraints from absence of Dirac-Misner strings and of conical defects leave only three independent parameters. This implies that, out of the four conserved charges of the solution, (M,Q,J_ψ,J_ϕ), at most only three of them, (M,Q,J_ψ), are independent. Eq. (4.14) shows that the dipole charge q is not an independent parameter and therefore there cannot be any continuous violation of uniqueness. This is in contrast with the situation when the net charge Q is zero, in which the dipole q is an independent parameter and so uniqueness is violated by a continuous parameter [2]. In the solutions in this paper, the addition of a charge, however small, implies that the net charge and the dipole charge must be related so as to avoid Dirac-Misner strings.

One can also argue that the solutions in this section do not exhibit discrete non-uniqueness, i.e.,that there are no two (or more) ring solutions which have the same four conserved charges.[4] To see this, fix the scale in the solutions by fixing the mass. Then define, like in [8], dimensionless quantities $j_{\psi,\phi}\propto J_{\psi,\phi}/\sqrt{G_5M^3}$, which characterize the spins

[4]There can be two solutions with the same values of the independent parameters, say (M,Q,J_ψ), but then they will be distinguished by the value of J_ϕ.

for fixed mass, and the (relative) energy above supersymmetry $m = [M - (3\pi/4G_5)Q]/M$. Note that m depends on α only. For fixed m, impose the balancing condition and the Dirac-Misner condition. Then there is only one free parameter, say μ, so one can use this to plot a curve in the (j_ψ, j_ϕ)-plane showing which values of j_ψ and j_ϕ are allowed. If this curve manages somehow to self-intersect then we would have two solutions with the same (m, j_ψ, j_ϕ), i.e.,discrete non-uniqueness. We have checked this for a large representative set of values of m, and found that the curve does not self-intersect, so uniqueness appears to hold among the rings in this section. Note, though, that we can expect charged spherical black holes of minimal supergravity to exist with the same conserved charges as some of these rings.

The solutions in this section do not admit any non-trivial supersymmetric limit to BPS rings. A natural conjecture is the existence of a five-parameter non-BPS ring solution, characterized by $(M, Q, J_\psi, J_\phi, q)$. This family would allow to describe thermal excitations above the BPS solution of [7], and presumably would exhibit continuous non-uniqueness, as in [2], through the parameter q. One would also expect discrete two-fold non-uniqueness for fixed parameters $(M, Q, J_\psi, J_\phi, q)$, at least for small enough values of Q and J_ϕ, by continuity to the solutions with $Q = 0 = J_\phi$, which are known to present this feature [2].

5. D1-D5-P black rings

Solutions where the three charges are interpreted as D1-D5-P charges are of particular relevance since, near supersymmetry, they will admit a dual description in terms of a rather well-understood 1+1 supersymmetric conformal field theory. Here we analyze the most general such solution obtainable with our methods and also two other important particular cases obtained by setting some dipoles or charges to zero. We denote by Q/q a solution with net charges Q and dipole charges q.

5.1 D1-D5-P/d1-d5-kkm black ring

The non-supersymmetric black ring with three charges, D1, D5 and momentum P, and dipole charges d1, d5, and Kaluza-Klein monopole kkm has metric in the string frame

$$ds^2_{\rm IIB} = d\tilde{s}^2_{\rm 5D} + \sqrt{\frac{h_2}{h_1}}\sqrt{\frac{H_1(y)H_2(x)}{H_1(x)H_2(y)}}\, d\mathbf{z}^2_{(4)} + \frac{h_3}{\sqrt{h_1h_2}}\frac{H_3(x)}{H_3(y)}\sqrt{\frac{H_1(y)H_2(y)}{H_1(x)H_2(x)}}\,[dz + A^3]^2\,, \quad (5.1)$$

where the non-vanishing components of the one-form A^3 are given in (A.7)–(A.9), and

$$\begin{aligned} d\tilde{s}^2_{\rm 5D} = &-\frac{1}{h_3\sqrt{h_1h_2}}\frac{F(y)}{F(x)}\sqrt{\frac{H_1(x)H_2(x)}{H_1(y)H_2(y)}}\,[dt + \omega_\psi(y)\,d\psi + \omega_\phi(x)\,d\phi]^2 + \\ &+\sqrt{h_1h_2}F(x)H_3(y)\sqrt{H_1(x)H_1(y)H_2(x)H_2(y)}\,\times \\ &\times\frac{R^2}{(x-y)^2}\left[-\frac{G(y)}{F(y)H(y)^3}\,d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)H(x)^3}\,d\phi^2\right]. \end{aligned} \quad (5.2)$$

The dilaton is

$$e^{2\Phi} = \frac{h_2}{h_1}\frac{H_1(y)H_2(x)}{H_1(x)H_2(y)}\,, \quad (5.3)$$

and the components of the RR 2-form potential $C^{(2)}$ are given in appendix B.

5.1.1 6D structure

KK dipole quantization. The solution (5.1) has a non-trivial structure along the sixth direction z due to the presence of the term $dz + A^3$. The quantization of the KK dipole can then be obtained easily by requiring regularity of the fibration. In order to eliminate the Dirac string singularity of A^3 at $x = +1$ we have to perform a coordinate (gauge) transformation $z \to \hat{z} - A^3_\phi(x = +1)$. The y-dependence here cancels if we impose the condition (2.22), and then the transformation is,

$$z \to \hat{z} = z - \frac{2R}{s_3}\left[\frac{C_2}{1-\mu_2}s_2c_1 + \frac{C_1}{1-\mu_1}c_2s_1\right]\phi\,. \tag{5.4}$$

With this, the geometry is free of Dirac singularities at $x = +1$. However, since z parametrizes a compact Kaluza-Klein direction, $z \sim z + 2\pi R_z$ the coordinate transformation (5.4) is globally well-defined only if

$$2\pi R_z = \pm\frac{2R}{n_{\rm KK}s_3}\left[\frac{C_2}{1-\mu_2}s_2c_1 + \frac{C_1}{1-\mu_1}c_2s_1\right]\Delta\phi \tag{5.5}$$

for some (positive) integer $n_{\rm KK}$. This condition gives rise to the KK monopole charge quantization:

$$q_{\rm KK} = \mp n_{\rm KK}R_z = -\frac{2R}{s_3}\left[\frac{C_2}{1-\mu_2}s_2c_1 + \frac{C_1}{1-\mu_1}c_2s_1\right]\frac{\Delta\phi}{2\pi}\,, \tag{5.6}$$

and then $q_{\rm KK} = q_3$ as given in (2.34).

Horizon geometry. Making use of the change of coordinates given by (2.36), it is clear that the five-dimensional part of the metric of the type-IIB solution is regular at $y_h = -1/\nu$. Note that the conformal factors multiplying the different terms in (5.1) remain finite and non-zero at y_h. In order to make the term $dz + A^3$ also regular at this point, we perform a change of coordinates

$$\begin{aligned} dz &= dz' + R(1+y)\left[\frac{C_\lambda}{F(y)}c_1c_2s_3 - \frac{C_1}{H_1(y)}c_1s_2c_3 - \frac{C_2}{H_2(y)}s_1c_2c_3 - \frac{C_3}{H_3(y)}s_1s_2s_3\right]\times \\ &\quad\times\frac{\sqrt{|F(y)|H(y)^3}}{G(y)}dy\,. \end{aligned} \tag{5.7}$$

In the (v,ψ') coordinates this term becomes manifestly analytic at y_h,

$$dz + A^3 = dz' + A^3_t dv + A^3_\psi d\psi' + A^3_\phi d\phi\,, \tag{5.8}$$

and imposing the charge quantization condition (5.6), it is perfectly regular on the horizon.

The horizon in six dimensions is a U(1) fibration over the $S^1 \times S^2$ geometry of the five-dimensional horizon (2.38). In the supersymmetric case [8], it was found that for certain values of the parameters the U(1) would Hopf-fiber over the S^2 to yield an S^3. In the present case, to find the same result we would have to perform the coordinate transformation

$$dz'' = dz' + A^3_\psi(x, y_h)\left(d\psi' + \frac{\omega_\phi(x)}{\omega_\psi(y_h)}d\phi\right) \tag{5.9}$$

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to eliminate the leg along the S^1 in the fiber in (5.8), but given the x-dependence this change is not compatible with the global periodicities of z and ϕ. Hence, the six-dimensional horizon of these rings is never globally of the form $S^1 \times S^3$. It may still be, though, that the more general non-supersymmetric solutions that we have conjectured to exist can actually have such a horizon geometry.

5.2 D1-D5/kkm black rings and two-charge supertubes

This is the simplest case of a two-charge black ring, with one dipole charge, that can be connected to a well-understood supersymmetric configuration in string theory. We will show that the supersymmetric limit of these black rings results into a two-charge supertube.

A particular case of these solutions (with all $\mu_i = 0$) was constructed in [3], but the supersymmetric limit of those rings could only yield supertubes with half the maximum angular momentum (for given charges and dipole).[5] Below we show how the inclusion of dipole parameters μ_i allows us to recover supertubes within the full range of allowed angular momenta. Hence we expect that the present solutions are sufficient to consistently describe the thermal excitations that keep $J_\phi = 0$ of supertubes with two charges, Q_1 and Q_2, and angular momentum J_ψ.

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5.2.1 Solution

In the general solution set $\alpha_3 = 0$ and $\mu_1 = \mu_2 = 0$. The string-frame metric for the D1-D5/kkm black ring is

$$\begin{aligned} ds^2 &= ds^2_{5\mathrm{D}} + \sqrt{\frac{h_2}{h_1}}\left(dz_1^2 + dz_2^2 + dz_3^2 + dz_4^2\right) + \\ &+ \frac{1}{\sqrt{h_1 h_2}}\frac{H_3(x)}{H_3(y)}\left[dz - R(1+x)\left(\frac{C_\lambda}{F(x)}s_1 s_2 - \frac{C_3}{H_3(x)}c_1 c_2\right)d\phi\right]^2, \end{aligned} \tag{5.10}$$

where

$$\begin{aligned} ds^2_{5\mathrm{D}} &= -\frac{1}{\sqrt{h_1 h_2}}\frac{F(y)}{F(x)}\left[dt + R(1+y)\left(\frac{C_\lambda}{F(y)}c_1 c_2 - \frac{C_3}{H_3(y)}s_1 s_2\right)d\psi\right]^2 + \\ &+ \sqrt{h_1 h_2}\, F(x) H_3(y)\, \frac{R^2}{(x-y)^2} \times \\ &\times \left[-\frac{G(y)}{F(y)H_3(y)}\,d\psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)H_3(x)}\,d\phi^2\right]. \end{aligned} \tag{5.11}$$

Note that now

$$h_i = 1 + \frac{(\lambda + \mu_3)(x-y)}{F(x)H_3(y)}\, {s_i}^2 \tag{5.12}$$

for $i = 1, 2$. The dilaton is

$$e^{2\Phi} = \frac{h_2}{h_1}\,, \tag{5.13}$$

[5] When comparing to the solutions in [3], the reader should be aware that a slightly different form of the seed is used, so the functions F, G and parameters λ, ν used below are not the same as those in [3].

and the non-vanishing components of the RR two-form potential are

$$C^{(2)}_{tz} = -\frac{(x-y)}{F(x)H_3(y)\,h_2}(\lambda+\mu_3)c_2 s_2\,, \tag{5.14}$$

$$C^{(2)}_{\psi z} = \frac{R(1+y)}{h_2}\frac{H_3(x)}{H_3(y)}\left[\frac{C_\lambda}{F(x)}c_1 s_2 - \frac{C_3}{H_3(x)}s_1 c_2\right], \tag{5.15}$$

$$C^{(2)}_{t\phi} = -\frac{R(1+x)}{h_2}\frac{F(y)}{F(x)}\left[\frac{C_\lambda}{F(y)}s_1 c_2 - \frac{C_3}{H_3(y)}c_1 s_2\right], \tag{5.16}$$

$$\begin{aligned} C^{(2)}_{\psi\phi} = & -\frac{R^2}{2}\sinh 2\alpha_1\left\{\frac{G(x)}{(x-y)}\frac{\lambda+\mu_3}{F(x)H_3(x)} + (1+x)\left(\frac{C_\lambda^2}{\lambda F(x)} + \frac{C_3^2}{\mu_3 H_3(x)}\right) + \right. \\ & \left. + \frac{1}{h_2}(1+x)(1+y)\frac{H_3(x)}{H_3(y)}\left[\frac{C_\lambda^2}{F(x)^2}s_2^2 + \frac{C_3^2}{H_3(x)^2}c_2^2\right]\right\} + \\ & + \frac{(1+x)(1+y)}{h_2}\frac{R^2}{2}\cosh 2\alpha_1 \sinh 2\alpha_2\,\frac{C_\lambda C_3}{F(x)H_3(y)}\,. \end{aligned} \tag{5.17}$$

Since $\omega_\phi = 0$ there are no dangerous Dirac-Misner strings in this solution.

5.2.2 Properties

The ADM mass of the black ring is

$$M = \frac{\pi R^2}{4G_5}\frac{1}{1-\nu}\{\lambda - \mu_3 + 2\lambda\mu_3 + (\lambda+\mu_3)(\cosh 2\alpha_1 + \cosh 2\alpha_2)\}\,, \tag{5.18}$$

and the angular momentum is

$$J_\psi = \frac{\pi R^3}{2G_5}\frac{\sqrt{(1-\lambda)(1+\mu_3)}}{(1-\nu)^2}\Big[(1+\mu_3)C_\lambda c_1 c_2 - (1-\lambda)C_3 s_1 s_2\Big]\,. \tag{5.19}$$

The D1- and D5 charges are

$$Q_{\rm D1} = \frac{R^2(\lambda+\mu_3)\sinh 2\alpha_2}{1-\nu}\,, \qquad Q_{\rm D5} = \frac{R^2(\lambda+\mu_3)\sinh 2\alpha_1}{1-\nu}\,, \tag{5.20}$$

and the dipole charge from the tubular Kaluza-Klein-monopole is[6]

$$q_{\rm KK} = 2R\left[\frac{C_\lambda}{1+\lambda}s_1 s_2 - \frac{C_3}{1-\mu_3}c_1 c_2\right]\frac{\Delta\phi}{2\pi}\,. \tag{5.21}$$

The horizon area is

$$\begin{aligned} \mathcal{A}_H = & \, 8\pi^2 R^3\,\frac{1}{(1-\nu)^2(1+\nu)}\Big|\epsilon_\lambda\sqrt{\lambda(1-\lambda^2)(\mu_3+\nu)}(1+\mu_3)c_1c_2 + \\ & + \epsilon_3\sqrt{\mu_3(1-\mu_3^2)(\lambda-\nu)}(1-\lambda)s_1s_2\Big|\,. \end{aligned} \tag{5.22}$$

[6] This is the same result as for q_3 in (2.34) but with the sign reversed so as to agree with the sign choices in [3, 8].

5.2.3 Supersymmetric limit: two-charge supertubes

Now take the supersymmetric limit $\alpha_1 \sim \alpha_2 \to \infty$ and $\mu_3 \sim \nu \sim \lambda \to 0$ keeping fixed

$$(\lambda + \mu_3)e^{2\alpha_1} = \frac{2Q_{\rm D5}}{R^2}\,, \qquad (\lambda + \mu_3)e^{2\alpha_2} = \frac{2Q_{\rm D1}}{R^2}\,. \tag{5.23}$$

Note that the dipole charge remains finite in the limit,

$$q_{\rm KK} = \frac{R}{2}\left(\epsilon_\lambda\sqrt{\lambda(\lambda-\nu)} - \epsilon_3\sqrt{\mu_3(\mu_3+\nu)}\right)e^{\alpha_1+\alpha_2}\,. \tag{5.24}$$

Taking the supersymmetric limit gives the string-frame metric

$$\begin{aligned} ds^2 &= -\frac{1}{\sqrt{h_1 h_2}}\left(dt + \frac{q_{\rm KK}}{2}(1+y)d\psi\right)^2 + \sqrt{h_1 h_2}\,d\mathbf{x}_4^2 + \\ &\quad + \frac{1}{\sqrt{h_1 h_2}}\left(dz - \frac{q_{\rm KK}}{2}(1+x)d\phi\right)^2 + \frac{h_2}{h_1}d\mathbf{z}_4^2 \end{aligned} \tag{5.25}$$

with

$$h_1 = 1 + \frac{Q_{\rm D5}}{2R^2}(x-y)\,, \qquad h_2 = 1 + \frac{Q_{\rm D1}}{2R^2}(x-y)\,. \tag{5.26}$$

The rest of the fields in the solution are also easily obtained.

This is exactly the 2-charge/1-dipole BPS solution of [8], which is the same as the two-charge supergravity supertube of [26, 6]. However, it is important to realize that if we approach the supersymmetric limit through a sequence of regular non-supersymmetric black rings, then the condition that they are balanced imposes restrictions on the parameters. Specifically, balancing the ring requires, in the limit, $2\nu = \lambda - \mu_3$. It is now convenient to define

$$k = \frac{2\mu_3}{\lambda + \mu_3}\,. \tag{5.27}$$

Since we must have $0 \le \mu_3 \le \lambda < 1$, k takes values between 0 and 1. Defining the function

$$f(k) = \frac{1}{2}\left(\epsilon_\lambda\sqrt{2-k} - \epsilon_3\sqrt{k}\right), \tag{5.28}$$

we can then write

$$q_{\rm KK} = \frac{\sqrt{Q_{\rm D1}Q_{\rm D5}}}{R}f(k)\,. \tag{5.29}$$

This can be used to eliminate R (which is not a proper invariant quantity of the supergravity solutions) from the angular momentum

$$J_\psi = \frac{\pi}{4G_5}R^2\,q_{\rm KK}\,, \tag{5.30}$$

and then express it in terms of the number of branes and Kaluza-Klein monopoles as[7]

$$J_\psi = \frac{N_{\rm D1}N_{\rm D5}}{n_{\rm KK}}\,{\rm sgn}[f(k)]f(k)^2\,. \tag{5.31}$$

[7] See e.g., [3, 8] for the expressions for the quantized brane numbers in terms of supergravity charges. Note that we take $n_{\rm KK}$ to be positive.

Using the four combinations of $\epsilon_\lambda, \epsilon_3 = \pm 1$, the values of the function $f(k)$ cover precisely the interval $[-1, 1]$ when k is varied between 0 and 1. Thus the supergravity solution yields exactly the range of angular momentum

$$|J_\psi| \leq \frac{N_{\rm D1} N_{\rm D5}}{n_{\rm KK}} \tag{5.32}$$

expected from the worldvolume supertube analysis [5, 6, 27, 28]. The solutions constructed earlier in [3] correspond instead to having $\mu_3 = 0$, hence $k = 0$ and $f^2(0) = 1/2$. Since we can now vary μ_3 and recover the whole range of expected values of the parameters, it seems that the solution above should be the most general non-supersymmetric black ring with two charges and one angular momentum.

The supersymmetric supertube solutions with angular momentum strictly below the bound (5.32) have naked singularities, while if the bound is saturated the solution is regular [29]. The saturation of the bound as a limit of our solutions is slightly subtle, since one must take $k = 1$ i.e.,$\mu_3 = \lambda$. For finite values of the parameters this would require $\nu = 0$, which corresponds to an extremal singular solution. So it would seem that in order to thermally deform the non-singular, maximally rotating supertube, one has to consider μ_3 strictly different from λ. This suggests that the energy gap of non-BPS excitations above the non-singular, maximally rotating supertube is larger than for under-rotating supertubes. Indeed this is expected, since in the former all the effective D1-D5 strings are singly wound (it is a ground state of the non-twisted sector of the dual CFT) and therefore the left- and right-moving open strings are energetically more expensive to excite.

Finally, on the issue of non-uniqueness, the solutions contain five parameters (i.e.,($\alpha_{1,2}$, μ_3, ν, λ, R), minus one constraint) but only four conserved charges $(M, Q_{1,2}, J_\psi)$, so we have a continuous one-parameter non-uniqueness. This is of course controlled by the dipole charge $q_{\rm KK}$. It is possible to see this explicitly by drawing plots of the area of the rings vs. J_ψ at different values of $q_{\rm KK}$, for fixed values of the mass and the charges. These curves look very similar to the ones for the dipole black ring with $N = 1$ in [2], so we shall not reproduce them here.

5.3 D1-D5-P/d1-d5 black ring: the black double helix

The supersymmetric configuration in the previous subsection is U-dual to a D1-P/d1 supergravity supertube. This is the supergravity description of a D1-helix: a D1-brane with a gyrating momentum wave, such that it coils around the ψ direction (hence the dipole d1) while carrying momentum along z. The supergravity solution is smeared along the direction of the tube [26]. T-dualizing along the internal T^4 directions one obtains a D5-P/d5 configuration, i.e.,a D5-helix.

The D1 and D5 can bind to form a supersymmetric double D1-D5 helix. To obtain a non-supersymmetric black D1-D5 helix (without a kkm tube), we must first demand

$$q_3 = 0 \qquad \text{and} \qquad q_1, q_2 \neq 0\,. \tag{5.33}$$

However, if we want the configuration to actually describe the excitations of a bound state of the D1-D5 helix, we must also require

$$q_1 Q_1 = q_2 Q_2 \tag{5.34}$$

even away from the supersymmetric state. Since Q_1 and q_2 (Q_2 and q_1) are associated to the number of windings of D5-branes (D1-branes) along z and along ψ, respectively, this equation is the condition that the pitches of the D1 and D5 helices are equal (see also [8]). The two helices can then bind and these black tubes are naturally interpreted as thermal excitations of a D1-D5 superhelix.

It is straightforward to see that the two conditions (5.33) and (5.34) imply

$$\mu_1 = \mu_2 = 0\,, \tag{5.35}$$

while all other parameters take non-zero values.

The supersymmetric limit of these solutions, as shown in section 3, correctly yields the BPS solutions with three charges and two dipoles of [8]. However, (5.35) implies that the two "constraint parameters" a and b in (3.4), (3.5), are not independent but satisfy $a^2 = 2b^2$. From (3.8) and (3.9) we obtain again the inequality (3.15), but now, instead of the bound (3.16), we find that

$$\frac{4G_5}{\pi}\,(J_\psi - J_\phi) = \frac{1}{2}\sqrt{\frac{Q_1 Q_2}{q_1 q_2}}\,(Q_3 - q_1 q_2)\,, \tag{5.36}$$

i.e.,we can only obtain half the maximum value for $J_\psi - J_\phi$. This restriction is reminiscent of the former situation for D1-D5/kkm supertubes in [3], and suggests that we need a still larger family of non-BPS solutions in order to describe the thermal deformations of supertubes with three charges and two dipoles and generic values of $J_\psi - J_\phi$.

5.4 Decoupling limit

The decoupling limit, relevant to $\mathrm{AdS}_3/\mathrm{CFT}_2$ duality of the D1-D5 system, is obtained by taking the string length to zero, $\alpha' = \ell_s^2 \to 0$, and then scaling the parameters in the solution in such a way that

$$\lambda\,, \quad \nu\,, \quad \mu_i\,, \quad \alpha_3\,, \quad \alpha' e^{2\alpha_{1,2}}\,, \quad \frac{R}{\alpha'}\,, \tag{5.37}$$

remain fixed. The (dimensionless) coordinates x, y also remain finite. Note in particular that the boosts associated to D5- and D1-brane charges become $\alpha_{1,2} \to \infty$, and that the energy of the excitations near the core of the solution is kept finite by scaling the parameter $R \sim \alpha'$. To keep the ten-dimensional string metric finite, we rescale it by an overall factor of α'.

The metric in the decoupling limit is of the same form as (5.1), with the functions F, G, H_i and h_3 unmodified, but $h_{1,2}$, ω and A^3 do change. One can see in general that the metric asymptotes to $\mathrm{AdS}_3 \times S^3 \times T^4$, but we shall provide details only for the two cases of interest of the previous subsections, with $\mu_1 = \mu_2 = 0$ (and in general $\alpha_3 \neq 0$), where the expressions get somewhat simplified. In this case one gets

$$h_{1,2} = \frac{Q_{1,2}}{2R^2}(1-\nu)\frac{x-y}{F(x)H_3(y)}\,, \tag{5.38}$$

and

$$\omega_\psi = \frac{G_5 J_\psi}{\pi R^2}\frac{2(1+y)}{F(y)H_3(y)}\frac{C_\lambda H_3(y) - C_3 F(y)}{C_\lambda(1+\mu_3) - C_3(1-\lambda)}\frac{(1-\nu)^2}{\sqrt{(1-\lambda)(1+\mu_3)}}\,,$$
$$\omega_\phi = \frac{G_5 J_\phi}{\pi R^2}\frac{1-x^2}{F(x)H_3(x)}\frac{(1-\nu)^2}{\sqrt{(1-\lambda)(1+\mu_3)}}\,, \tag{5.39}$$

while the expression for A^3 is not comparatively simpler. The asymptotic region is again at $x \to y \to -1$. If we perform the change of coordinates

$$r^2 = R^2\frac{1-x}{x-y}\frac{(1-\lambda)(1+\mu_3)}{1-\nu}\,, \qquad \cos^2\theta = \frac{1+x}{x-y}\,, \tag{5.40}$$

introduce canonical angular variables (2.19), and gauge-transform so that A^3_t vanishes at infinity, then the asymptotic metric at $r \to \infty$ is

$$ds^2_{\text{IIB}} \to \frac{r^2}{\sqrt{Q_1 Q_2}}\left(-dt^2 + dz^2\right) + \sqrt{Q_1 Q_2}\frac{dr^2}{r^2} + \sqrt{Q_1 Q_2}\left(d\theta^2 + \sin^2\theta d\tilde{\psi}^2 + \cos^2\theta d\tilde{\phi}^2\right) +$$
$$+\sqrt{\frac{Q_2}{Q_1}}d\mathbf{z}^2_{(4)}\,, \tag{5.41}$$

i.e.,the correct asymptotic geometry of $\mathrm{AdS}_3 \times S^3 \times T^4$.

Away from the asymptotic boundary the geometry does not factorize into a product space $\mathrm{AdS}_3 \times S^3 \times T^4$, not even locally. In general, the geometry does not appear to become simple even near the horizon. This is in contrast to what was found in the supersymmetric case with three charges and three dipoles, where near the horizon a second, different factorization into (locally) $\mathrm{AdS}_3 \times S^3 \times T^4$ happens [8, 12]. With three charges and only two dipoles, this factorization does not happen even near the core of the supersymmetric solutions. But perhaps the yet to be found non-BPS solution that can retain all three dipoles in the supersymmetric limit will show this property.

6. Discussion

We have shown how to construct non-supersymmetric three-charge black rings via boosts and dualities, overcoming the problems previously encountered in ref. [3]. Technically, the main issue is the requirement that the one-form ω associated to the rotation of the ring be well-defined (absence of Dirac-Misner strings). This was also an important ingredient when constructing the supersymmetric rings by solving the supersymmetry-preservation equations of [30]. In the present case, we have achieved it by having more parameters, associated to dipole charges, in our seed solution.

We expect that a larger family of non-supersymmetric black rings with nine-parameters $(M, J_\psi, J_\phi, Q_{1,2,3}, q_{1,2,3})$ exists, and that the general solutions of [7]–[10] are recovered in the supersymmetric limit. The nine parameters would yield three-fold continuous non-uniqueness, furnished by the non-conserved dipole charges $q_{1,2,3}$. This is like in the dipole ring solutions in [2] but larger than the two-fold continuous non-uniqueness of the supersymmetric rings of [8]. Presumably, one of the additional parameters in the conjectured larger family of solutions will not describe proper near-supersymmetric excitations of the supertube. Instead they should be interpreted as the presence of dipole strings or branes in the configuration that are not bound to the supertube but rather superimposed on it. Since these dipole branes break all supersymmetries, they must disappear in the BPS limit.

The extra parameters that are still missing from our current solutions should provide the freedom to vary the angular momentum J_ϕ and all the three dipoles independently of other parameters. In particular, we expect that black rings with two charges should exist that carry both angular momenta J_ψ and J_ϕ. Note, though, that in the supersymmetric limit J_ϕ disappears, which is consistent with the fact that J_ϕ is expected to be carried by the coherent polarization of (R-charged) left- and right-moving fermionic open string excitations. In our solutions the total macroscopic J_ϕ must vanish. So the two-charge solutions in this paper can describe the non-BPS excitations that carry total $J_\phi = 0$.[8] This should be already enough to address in detail the issues of black hole non-uniqueness and near-supersymmetric black ring entropy from a microscopic viewpoint, along the lines proposed in ref. [3]. We hope to tackle these problems in the future.

The solutions with three charges and two dipoles in this paper are similarly expected to describe the non-BPS excitations that do not add to the J_ϕ of a D1-D5-P/d1-d5 super-helix. However, in this case we seem to be restricted to considering only excitations above the supersymmetric state with half the maximum value of $J_\psi - J_\phi$. This is analogous to the former situation in [3], and once again points to the need to find a larger family of solutions. Note, however, that by including more general excitations our solutions do describe thermally deformed D1-D5-P/d1-d5 supertubes with any value of $J_\psi - J_\phi$ in the permitted range (3.17). As we have seen, these solutions are free of pathologies and are regular on and outside the horizons.

One can speculate about generalizations of the non-supersymmetric black rings. Refs. [9, 14] constructed supersymmetric ring solutions with arbitrary cross-sections. However, it was argued in [31] that unless the cross-section is circular these rings are not truly black holes, since they do not have smooth horizons. Adding energy to a ring with a non-circular cross-section is unlikely to yield a stationary black ring, since the lumpiness of the rotating ring will presumably cause the system to radiate when it is not supersymmetric.

Finally, a particularly interesting spin-off of our study is the evidence we have found in favor of the proposal in ref. [32] for one criterion to admit naked singularities in supergravity solutions. The supersymmetric limits of our black rings do indeed typically result in solutions with naked singularities. Ref. [32] proposed that solutions which admit thermal deformations, i.e.,arise as the zero-temperature limit of black holes with a regular horizon, should be regarded as physically sensible. This is precisely what we have found, and in a very non-trivial manner. Taking the supersymmetric limit of our rings results in supertubes with parameters precisely within the ranges that were earlier determined using entirely different criteria — worldvolume theory constraints (including unitarity), and absence of localized causal violations in BPS solutions. Solutions with sick worldvol-

[8] We could have two-charge supertubes with $J_\phi \neq 0$ if $\mu_1, \mu_2 \neq 0$. But then $q_1, q_2 \neq 0$, so these excitations take the system further away from the supersymmetric state.

ume theories, or spacetime causal pathologies, such as over-rotating supertubes, do not admit thermal deformations. This is a remarkable consistency check of the low energy supergravity description of string theory configurations.

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A. Three-form fields for the 11D solution

We here give explicitly the non-zero components of the three-form potential for the eleven-dimensional solution in section 2:

$$A^1_t = \frac{U_1-1}{h_1}c_1s_1\,, \tag{A.1}$$

$$A^1_\psi = \frac{R(1+y)}{h_1}\left[U_1\frac{C_\lambda}{F(y)}s_1c_2c_3 - U_1\frac{C_1}{H_1(y)}s_1s_2s_3 - \frac{C_2}{H_2(y)}c_1c_2s_3 - \frac{C_3}{H_3(y)}c_1s_2c_3\right], \tag{A.2}$$

$$A^1_\phi = -\frac{R(1+x)}{h_1}\left[\frac{C_\lambda}{F(x)}c_1s_2s_3 - \frac{C_1}{H_1(x)}c_1c_2c_3 - U_1\frac{C_2}{H_2(x)}s_1s_2c_3 - U_1\frac{C_3}{H_3(x)}s_1c_2s_3\right], \tag{A.3}$$

$$A^2_t = \frac{U_2-1}{h_2}c_2s_2\,, \tag{A.4}$$

$$A^2_\psi = \frac{R(1+y)}{h_2}\left[U_2\frac{C_\lambda}{F(y)}c_1s_2c_3 - \frac{C_1}{H_1(y)}c_1c_2s_3 - U_2\frac{C_2}{H_2(y)}s_1s_2s_3 - \frac{C_3}{H_3(y)}s_1c_2c_3\right], \tag{A.5}$$

$$A^2_\phi = -\frac{R(1+x)}{h_2}\left[\frac{C_\lambda}{F(x)}s_1c_2s_3 - U_2\frac{C_1}{H_1(x)}s_1s_2c_3 - \frac{C_2}{H_2(x)}c_1c_2c_3 - U_2\frac{C_3}{H_3(x)}c_1s_2s_3\right], \tag{A.6}$$

$$A^3_t = \frac{U_3-1}{h_3}c_3s_3\,, \tag{A.7}$$

$$A^3_\psi = \frac{R(1+y)}{h_3}\left[U_3\frac{C_\lambda}{F(y)}c_1c_2s_3 - \frac{C_1}{H_1(y)}c_1s_2c_3 - \frac{C_2}{H_2(y)}s_1c_2c_3 - U_3\frac{C_3}{H_3(y)}s_1s_2s_3\right], \tag{A.8}$$

$$A^3_\phi = -\frac{R(1+x)}{h_3}\left[\frac{C_\lambda}{F(x)}s_1s_2c_3 - U_3\frac{C_1}{H_1(x)}s_1c_2s_3 - U_3\frac{C_2}{H_2(x)}c_1s_2s_3 - \frac{C_3}{H_3(x)}c_1c_2c_3\right], \tag{A.9}$$

where the functions U_i are defined as

$$U_i = \frac{F(y)H(x)^3}{F(x)H(y)^3}\frac{H_i(y)^2}{H_i(x)^2}\,. \tag{A.10}$$

B. The RR two-form potentials for the D1-D5-P black ring solution

We here give the expressions for the non-zero components of the Ramond-Ramond two-form potential of the D1-D5-P non-supersymmetric black ring solution of type-IIB supergravity

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given in section 5:

$$C_{tz}^{(2)} = \frac{U_2-1}{h_2} c_2 s_2 \,, \tag{B.1}$$

$$C_{\psi z}^{(2)} = \frac{R(1+y)}{h_2}\left[U_2 \frac{C_\lambda}{F(y)} c_1 s_2 c_3 - \frac{C_1}{H_1(y)} c_1 c_2 s_3 - U_2 \frac{C_2}{H_2(y)} s_1 s_2 s_3 - \frac{C_3}{H_3(y)} s_1 c_2 c_3\right], \tag{B.2}$$

$$C_{\phi z}^{(2)} = -\frac{R(1+x)}{h_2}\left[\frac{C_\lambda}{F(x)} s_1 c_2 s_3 - U_2 \frac{C_1}{H_1(x)} s_1 s_2 c_3 - \frac{C_2}{H_2(x)} c_1 c_2 c_3 - U_2 \frac{C_3}{H_3(x)} c_1 s_2 s_3\right], \tag{B.3}$$

$$C_{t\psi}^{(2)} = \frac{R(1+y)}{h_2}\left[U_2 \frac{C_\lambda}{F(y)} c_1 s_2 s_3 - \frac{C_1}{H_1(y)} c_1 c_2 c_3 - U_2 \frac{C_2}{H_2(y)} s_1 s_2 c_3 - \frac{C_3}{H_3(y)} s_1 c_2 s_3\right], \tag{B.4}$$

$$C_{t\phi}^{(2)} = -\frac{R(1+x)}{h_2}\left[\frac{C_\lambda}{F(x)} s_1 c_2 c_3 - U_2 \frac{C_1}{H_1(x)} s_1 s_2 s_3 - \frac{C_2}{H_2(x)} c_1 c_2 s_3 - U_2 \frac{C_3}{H_3(x)} c_1 s_2 c_3\right], \tag{B.5}$$

$$\begin{aligned}
C_{\psi\phi}^{(2)} = & -R^2 c_1 s_1 \Bigg\{ \frac{G(x)}{(x-y)} \left(\frac{\lambda+\mu_3}{F(x)H_3(x)} + \frac{\mu_2-\mu_1}{H_1(x)H_2(x)}\right) + \\
& \qquad + \frac{1}{h_2}(1+x)(1+y)\Bigg[U_2 \frac{C_\lambda^2}{F(x)F(y)} s_2^2 - \frac{C_1^2}{H_1(x)H_1(y)} c_2^2 - \\
& \qquad\qquad - U_2 \frac{C_2^2}{H_2(x)H_2(y)} s_2^2 + \frac{C_3^2}{H_3(x)H_3(y)} c_2^2\Bigg] + \\
& \qquad + (1+x)\left(\frac{C_\lambda^2}{\lambda F(x)} - \frac{C_1^2}{\mu_1 H_1(x)} + \frac{C_2^2}{\mu_2 H_2(x)} + \frac{C_3^2}{\mu_3 H_3(x)}\right)\Bigg\} + \\
& + \frac{(1+x)(1+y)}{h_2} R^2 c_2 s_2 \Bigg\{ \frac{C_\lambda C_3}{F(x)H_3(y)} s_1^2 + U_2 \frac{C_\lambda C_3}{F(y)H_3(x)} c_1^2 + \\
& \qquad + U_2 \frac{C_1 C_2}{H_1(x)H_2(y)} s_1^2 + \frac{C_1 C_2}{H_1(y)H_2(x)} c_1^2 \Bigg\} \,,
\end{aligned} \tag{B.6}$$

where U_i were defined in (A.10).

C. Limit of spherical black hole

We can take a limit of our solutions to recover non-supersymmetric spherical black holes with three charges. This limit is actually the same as described in [2], and involves taking $\lambda, \nu \to 1$ and $R \to 0$ while keeping finite the parameters a, m, defined as

$$m = \frac{2R^2}{1-\nu} \,, \qquad a^2 = 2R^2 \frac{\lambda-\nu}{(1-\nu)^2} \,. \tag{C.1}$$

The coordinates x, y degenerate in this limit, so we introduce new ones, r, θ, through

$$\begin{aligned}
x &= -1 + 2\left(1 - \frac{a^2}{m}\right) \frac{R^2 \cos^2\theta}{r^2 - (m-a^2)\cos^2\theta} \,, \\
y &= -1 - 2\left(1 - \frac{a^2}{m}\right) \frac{R^2 \sin^2\theta}{r^2 - (m-a^2)\cos^2\theta} \,,
\end{aligned} \tag{C.2}$$

and rescale ψ and ϕ

$$(\psi, \phi) \to \sqrt{\frac{m - a^2}{2R^2}}\, (\psi, \phi) \tag{C.3}$$

so they have canonical periodicity 2π. Then we recover the metric

$$\begin{aligned} ds^2_{5\mathrm{D}} \;=\; & -(h_1 h_2 h_3)^{-2/3} \left(1 - \frac{m}{\Sigma}\right) \left(dt - \frac{ma \sin^2\theta}{\Sigma - m}\, c_1 c_2 c_3\, d\psi - \frac{ma \cos^2\theta}{\Sigma}\, s_1 s_2 s_3\, d\phi\right)^2 + \\ & +(h_1 h_2 h_3)^{1/3} \left[\Sigma \left(\frac{dr^2}{\Delta} + d\theta^2\right) + \frac{\Delta \sin^2\theta}{1 - m/\Sigma}\, d\psi^2 + r^2 \cos^2\theta\, d\phi^2\right], \\ & \Delta \equiv r^2 - m + a^2\,, \qquad \Sigma \equiv r^2 + a^2 \cos^2\theta\,, \qquad h_i = 1 + \frac{m s_i^2}{\Sigma}\,. \end{aligned} \tag{C.4}$$

This is the particular case of the rotating black hole with three charges in [22] that is obtained by setting one of the two rotation parameters of the initial seed black hole to zero.

Note that we have not prescribed any limiting value for the parameters μ_i. As was the case for the dipole rings in [2], in the limit all the functions $H_i(\xi) \to 1 + \mu_i$ become constants that can be absorbed in rescalings of the coordinates. Then the limiting spherical black hole solution is actually independent of these parameters and therefore they cannot provide it with any kind of 'hair'.

D. Infinite radius limit

In the limit where the radius of the S^1 of the ring becomes infinite the ring becomes a black string carrying momentum along its length. The dipole charges become conserved charges, so in this limit we obtain a five-dimensional black string with six charges and momentum. Reduction to four dimensions along the length of the string results in a non-supersymmetric four-dimensional black hole with seven charges. Their extremal limit in this case is also a supersymmetric limit.

We take $R \to \infty$ while keeping fixed

$$r = -\frac{R}{y}\,, \qquad \cos\theta = x\,, \qquad \eta = R\psi\,. \tag{D.1}$$

In order to get a finite limit, we also take $\lambda, \nu, \mu_i \to 0$ keeping fixed

$$r_0 = \nu R\,, \qquad r_0 \cosh^2\sigma = \lambda R\,, \qquad r_0 \sinh^2\gamma_i = \mu_i R\,. \tag{D.2}$$

Note first that the balancing condition gives

$$\sinh^2\sigma = 1 + \sum_{i=1}^{3} \sinh^2\gamma_i \tag{D.3}$$

and the Dirac-Misner condition (2.22) becomes

$$0 = \omega_\phi = \epsilon_\lambda \sinh 2\sigma s_1 s_2 s_3 - \epsilon_1 \sinh 2\gamma_1 s_1 c_2 c_3 - \epsilon_2 \sinh 2\gamma_2 c_1 s_2 c_3 - \epsilon_3 \sinh 2\gamma_3 c_1 c_2 s_3\,. \tag{D.4}$$

The metric becomes

$$
\begin{aligned}
ds^2_{11\text{D}} = & -\frac{1}{(h_1 h_2 h_3)^{2/3}} \frac{\hat{f}}{(\hat{h}_1 \hat{h}_2 \hat{h}_3)^{1/3}} \Big[dt + \omega_\eta \, d\eta\Big]^2 + \\
& + (h_1 h_2 h_3)^{1/3} \left\{ \frac{f}{\hat{f} (\hat{h}_1 \hat{h}_2 \hat{h}_3)^{1/3}} d\eta^2 + (\hat{h}_1 \hat{h}_2 \hat{h}_3)^{2/3} \left(\frac{dr^2}{f} + r^2 d\Omega_2^2 \right) \frac{1}{(\hat{h}_1 \hat{h}_2 \hat{h}_3)^{1/3}} \times \right. \\
& \left. \times \left[\frac{\hat{h}_1}{h_1} (dz_1^2 + dz_2^2) + \frac{\hat{h}_2}{h_2} (dz_3^2 + dz_4^2) + \frac{\hat{h}_3}{h_3} (dz_5^2 + dz_6^2) \right] \right\},
\end{aligned}
\tag{D.5}
$$

where

$$
f = 1 - \frac{r_0}{r}, \qquad \hat{f} = 1 - \frac{r_0 \cosh^2 \sigma}{r}, \qquad \hat{h}_i = 1 + \frac{r_0 \sinh^2 \gamma_i}{r}, \tag{D.6}
$$

and

$$
\begin{aligned}
\omega_\eta = & -\frac{r_0}{2r} \Big[\hat{f}^{-1} \epsilon_\lambda \sinh 2\sigma c_1 c_2 c_3 - \hat{h}_1^{-1} \epsilon_1 \sinh 2\gamma_1 c_1 s_2 s_3 - \\
& - \hat{h}_2^{-1} \epsilon_2 \sinh 2\gamma_2 s_1 c_2 s_3 - \hat{h}_3^{-1} \epsilon_3 \sinh 2\gamma_3 s_1 s_2 c_3 \Big].
\end{aligned}
\tag{D.7}
$$

Further, using the balancing condition we have

$$
h_i = 1 + \frac{2\, r_0 \, \hat{h}_i {s_i}^2}{r \, \hat{h}_1 \hat{h}_2 \hat{h}_3} \left[\sinh^2 \sigma - \sinh^2 \gamma_i + \frac{r_0}{2r} \left(\cosh^2 \sigma \sinh^2 \gamma_i + \frac{\prod_{j=1}^3 \sinh^2 \gamma_j}{\sinh^2 \gamma_i} \right) \right]. \tag{D.8}
$$

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